Equilibrium States of Torques

Your Name ______________________________

Partners' Names ______________________________

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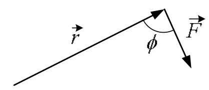
TA's signature allowing you to take the quiz or leave ______________

Obtained reasonable experimental results?	yes ☐
Answered questions?	yes ☐
Cleaned your table?	yes ☐

Introduction

The torque can be explained as "force" associated with rotational motions. The exact definition of torque, τ, is the force, F, exerting at a point times displacement, r, from a pivot to the point:

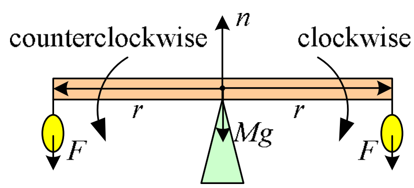


$$\tau = Fr\sin\phi$$

The dimension of torque is [M][L^2][T^{-2}]. The unit is N·m. The torque also depends on the angles between the force vector and the displacement (lever arm), ϕ, as shown. When the angle is 90 degrees, sin 90^o = 1. Therefore, the above formula can be $\tau = Fr$, which becomes the maximum torque.

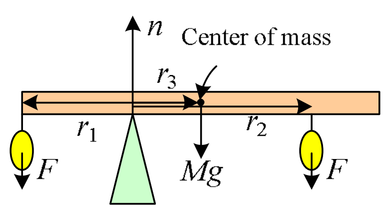
Any object can be reduced into one point to describe its linear motion. This point is called the center of mass (or center of gravity). If the mass is uniformly distributed, the center of mass has to be the geometrical center. Gravitational force acts on the center of mass.

When an object is at equilibrium under a rotational condition, both net torque and net force will become zero. The following is an example:



The lever arm and the forces are perpendicular each other. There are four forces, F, n (normal force), Mg (weight of the stick), and F again. The sum of these vectors must be zero. The torques for the clockwise and counterclockwise about fulcrum is equal as the magnitude and opposite as the directions, which makes the sum of torques zero. (Note that the forces at the pivot do not create torque since there is no lever arm for them.)

Here is another case. The pivot is shifted from the center of mass; then, the gravitational force at the point also creates a torque this time. This also holds both zero net force and zero net torque due to equilibrium. Considering the direction of each force and torque, we can have the following relationships:

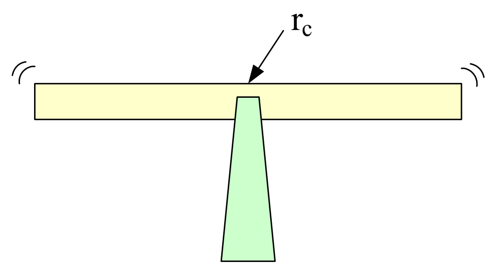


The net force holds $n - F - F - Mg = 0$; and the net torque does $-Fr_2 - Mgr_3 + Fr_1 = 0$. (Note that the normal force does not create its torque.)

Objectives:

- To learn concepts of the center of gravity (or center of mass)
- To verify the conditions of equilibrium state of a rotational body
- To apply the theory to measure mass of an object (a meter stick)

1. Center of Mass



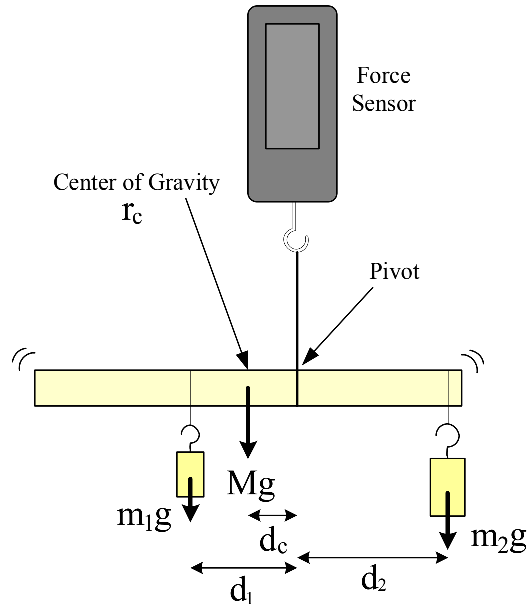
Find the center of mass of a meter stick.
Obtain the balance of a meter stick without hanging mass.

Reading of the center of mass r_c = ________________ (   ) ← units

Weigh the meter stick with a balance.
Before you do this, take off the metal apparatus (hinge) from the meter stick.

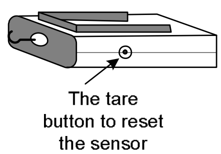
Mass of meter stick (by weighing with a balance) M = _______________ (   ) ← units **(1)**

2. Conditions of Equilibrium



Use the force sensor, and set up as shown. M is mass of the meter stick. For the DataStudio, start up the software; and click “Create Experiment.” Click on the analogue channel where the force sensor is connected. Select the “Force Sensor.” For the display, select “Digits.” [Note: The force sensor indicates a negative number. This is because it is pulled by the string. However, we are interested in the force on the stick which has the opposite direction. Thus, use the positive value for the tension to plug in the following equation.]

✶ Note that you have to reset the force sensor by pressing the tare button.



- The net external force

$$\sum F = -m_1 g - Mg - m_2 g + \text{tension (in force sensor)} =$$

(Calculate here.)

______________ (   ) ← units

- The net external torque [Note that the tension does not create the torque.]

$$\sum \tau = \tau_{\text{pivot}} + \tau_{\text{gravity}} + \tau_{m_1} + \tau_{m_2}$$
$$= 0 + Mgd_c + m_1 g d_1 - m_2 g d_2 =$$

(Calculate here.)

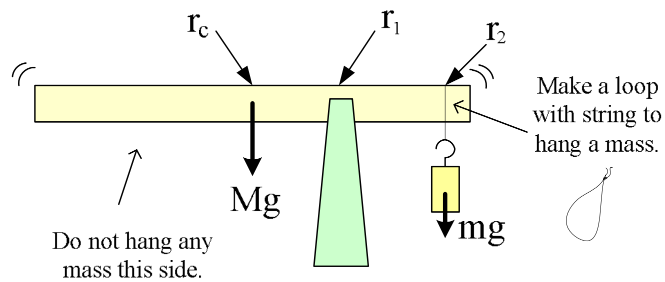
______________( ) ← units

Question 1
Are the net force and torque close to zero? Explain why it is so.

3. Application of equilibrium of torques (*Use different hanging masses for each trial.*)

- **Theory**

The positions, r_1, r_2, and r_c are the fulcrum, hanging mass, and center of the mass of the meter stick.



The torque due to mass of the stick is:
$\tau_M = Mg|r_c - r_1|\sin\theta$. If the stick is <u>completely horizontal</u>, the angle between the gravitational force and the lever arm is 90°; namely, $\sin\theta$ becomes 1.

The torque by the hanging mass is:
$\tau_m = mg|r_1 - r_2|\sin\theta$. With the same reason, $\sin\theta$ becomes 1. These two torques are at equilibrium; namely, they are equal,

$Mg|r_c - r_1| = mg|r_1 - r_2|$. Solving for mass of the meter stick, we obtain $M = \dfrac{|r_1 - r_2|}{|r_c - r_1|}m$

- **Procedure:**

Important Tips:
- Use SI units. (meters, and kilograms)
- The range of r_1 should be from 0.25m to 0.75m. (If you want a challenge, go beyond.)
- At equilibrium, the meter stick **<u>must be horizontal</u>**.
- Try to read 4 significant figures for the meter stick calibration.

Try six different r_1's.
Change the positions of fulcrum six times. Also change the hanging mass for each trial.

Calculate $M = \frac{|r_1 - r_2|}{|r_C - r_1|}m$ for each case, and the average; then, obtain the standard deviation.

r_1, r_2, and r_c are <u>just reading</u> from the meter stick.

m (hanging mass)	r_1	r_2	$\|r_1 - r_2\|$	$\|r_c - r_1\|$	$M = \frac{\|r_1 - r_2\|}{\|r_c - r_1\|}m$

Average and standard deviation: $M = ($ ±) [ ]$\Leftarrow$ unit

Question 2

How does this compare with the mass obtained by weighing (1) the meter stick?

The mass obtained by equilibrium of torques is equal to the mass by weighing with a balance. If your results are off, discuss the causes of error.

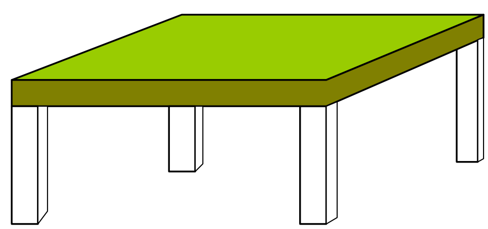
Question 3

Deduce whether accuracy is improved by choosing large or small value of m, $|r_1\text{-}r_2|$ or $|r_c\text{-}r_1|$.

Comparing the agreement of individual measurements with the average and with the result of weighing the meter stick, deduce whether accuracy is improved by choosing large or small value of m, $|r_1 - r_2|$ or $|r_c - r_1|$. Discuss this from your experimental results.

Question you want to explore

(Discuss with your partners or TA. If your TA suggests, address this discussion to your lab report.)



This lab shows that the torque balance derives the mass of an object. Suppose you have a large object such as a sturdy table shown. **How do you measure the mass <u>without any scale or weight measurer</u>?** Here are a few things to discuss this question.

- By following this lab procedure (in the third part), what tools would you use to obtain table's mass? Describe the each procedure.
- Under what conditions would it be more difficult to measure a mass by using this method? Does it depend on the shape, mass, place, or material of the object?
- What are the difference and similarity between the principle of this method and the mechanism of a beam balance?