Rotational Kinetic Energy and Moment of Inertia

Your Name ______________________________

Partners' Names ______________________________

____

TA's signature allowing you to take the quiz or leave ____________

Obtained reasonable experimental results?	yes ☐
Answered questions?	yes ☐
Cleaned your table?	yes ☐

Introduction

Linear motion of kinetic energy (*KE*) is given in the previous labs as

$$KE = \frac{1}{2}mv^2$$

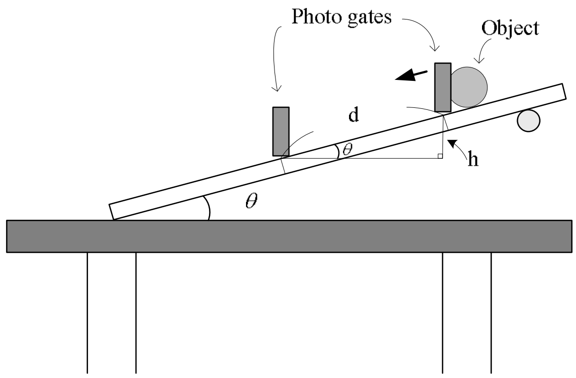
This is actually for an object which is not rotating. When the object rotates, the rotational kinetic energy (*RKE*) has to be taken into account:

$$RKE = \frac{1}{2}I\omega^2$$

where I is moment of inertia, and ω is the angular speed. Namely, total kinetic energy consists of both linear and rotational energies in general. Obviously, the total energy also has to be conserved with the potential energy (*PE*) of the system unless there is a work done by external forces. Namely, the conservation of energy gives the following:

$$KE + RKE = PE$$

Rotational motion has an independent property to be added to a general motion. The unit of rotational kinetic energy is also joules (J).



According to the experimental set up in the figure, due to gravity, the average velocity, $\bar{v}$, can be $(v_f+v_i)/2$. The initial velocity is made to be zero, so $\bar{v} = v_f/2$. Thus, the final velocity, v_f, is solved as $2\bar{v}$. Using the initial velocity of the object is zero; we have the explicit expression of the conservation of energy:

$$\frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega^2 = Mgh$$

The angular velocity is given by the radius of the object and its tangential velocity, $\omega = \frac{v_f}{R}$.
Then, the theoretical final velocity of an object is obtained as follows:

$$v_{\text{f, theoretical}} = \sqrt{\frac{2gh}{1+\frac{I}{MR^2}}}. \qquad \textbf{(The formula)}$$

where M is the mass,; and R is the radius; I is the moment of inertia of an object; and h is the vertical rolling distance, starting from rest as shown in the above figure.

Objectives:

- To extend conservation of energy by including rotational kinetic energy
- To verify the above theory by measuring the final velocity of various objects

- **Question before the experiment:**

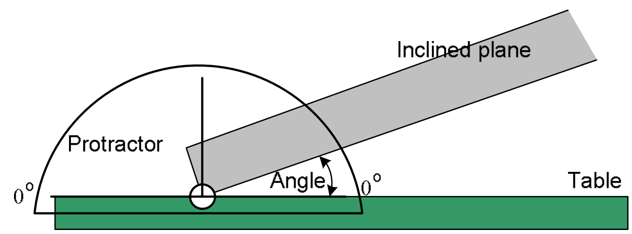
By remembering the previous lab, explain the difference between linear inertia and rotational inertia in layman's terms.

The Basic Set up

Start up DataStudio. Click "Create Experiment." Click on the digital channel of the interface, and select "photo gate." Repeat the same thing for the second photo gate. For the display, select "Table." Choose "Time Between Any Gates." Record the "Elapsed Time."

① Measure the rolling distance along ramp and the inclined angle.

The distance must be between photo gates. More precisely, it is between the beams of infrared red. The angle is from the surface of table to the bottom of the plate. Use a protractor. **Note: The angle will be changed when the track moves horizontally!**



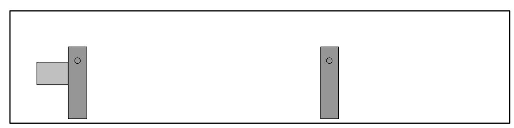
② Calculate the vertical distance, *h*.

From the figure, it is obtained by $h = d\sin\theta$.

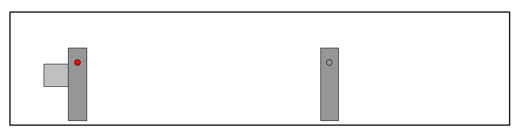
③ Measure the rolling time for the distance.

Since it must start from rest, the following instruction is very important. (Note: The figures are the top view.)

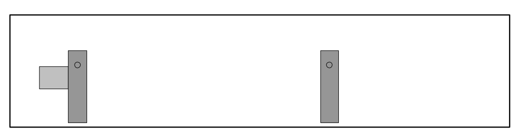
Put the object near the first gate.



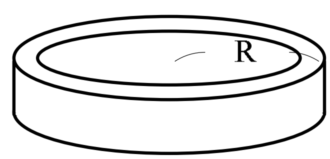
When it crosses the beam, the light on the gate will flash.



After you get the object back a little where the object does not cross the photo gate, click start and release the object.



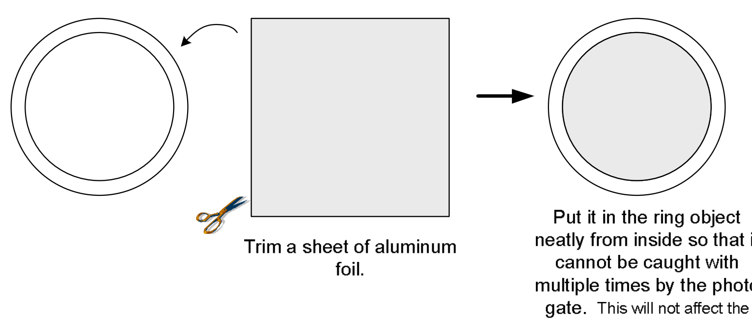
1. Velocity of a rolling object I (Ring)



Mass M = ____ (kg)

Radius R = Diameter ÷ 2 = ____ (m)

Theoretical moment of inertia $I = MR^2$ = ______________________________ $(\text{kg}\cdot\text{m}^2)$



Angle, θ = ____ (^{o})

Rolling distance along ramp, d = __________________________________ (m)

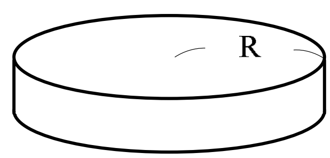
Vertical distance, $h = d \sin\theta$ = ____________________________________ (m)

t Time between photo gates	$\bar{v} = d/t$ Average velocity	$v_f = 2\bar{v}$ Final velocity (Experimental)	$v_{\mathrm{f,theoretical}}$ Use (The formula).	% difference $\frac{\lvert v_f - v_{\mathrm{f,theoretical}} \rvert}{\frac{1}{2}(v_f + v_{\mathrm{f,theoretical}})} \times 100\%$

Question 1-1: Are the experimental and theoretical values close each other?

Question 1-2: Is there any energy loss in this experiment? (*If some of the energy is lost, the experimental final velocity will be below the theoretically predicted values. If the experimental final velocity is larger, what is the significant reason? You can't create energy from nothing!*)

2. Velocity of a rolling object II (Disk)



Please rip off the last page of this data sheet, and look at the formulae to calculate the following.

Mass M = ____ (   ) ← units

Radius R = Diameter ÷ 2 = ____ (   ) ← units

Theoretical moment of inertia $I = \frac{1}{2}MR^2$ = ____________________________ (   ) ← units

Angle, θ = ____ (   ) ← units

Rolling distance along ramp, d = __________________________________ (   ) ← units

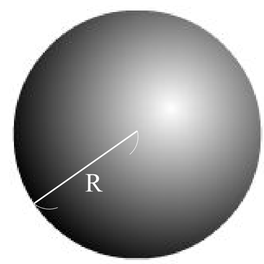
Vertical distance, $h = d \sin\theta$ = ____________________________________ (   ) ← units

t Time between photo gates	$\bar{v} = d/t$ Average velocity	$v_f = 2\bar{v}$ Final velocity (Experimental)	$v_{\text{f,theoretical}}$ Use (The formula).	% difference $\frac{\lvert v_f - v_{\text{f,theoretical}} \rvert}{\frac{1}{2}(v_f + v_{\text{f,theoretical}})} \times 100\%$

Question 2-1: Are the experimental and theoretical values close each other?

Question 2-2: Is there any energy loss in this experiment? (*If some of the energy is lost, the experimental final velocity will be below the theoretically predicted values. If the experimental final velocity is larger, what is the significant reason? You can't create energy from nothing!*)

3. **Velocity of a rolling object III <u>(Sphere)</u>**



Mass M = ____ (   ) ← units

Radius R = Diameter ÷ 2 = _________________________________ (   ) ← units

Theoretical moment of inertia $I = \frac{2}{5}MR^2$ = ____________________ (   ) ← units

Angle, θ = _______________________________________ (   ) ← units

Rolling distance along ramp, d = _________________________________ (   ) ← units

Vertical distance, $h = d\sin\theta$ = ___________________________________ (   ) ← units

t Time between photo gates	$\bar{v} = d/t$ Average velocity	$v_f = 2\bar{v}$ Final velocity (Experimental)	$v_{\text{f,theoretical}}$ Use (The formula).	% difference $\frac{\lvert v_f - v_{\text{f,theoretical}} \rvert}{\frac{1}{2}(v_f + v_{\text{f,theoretical}})} \times 100\%$

Question 3-1: Are the experimental and theoretical values close each other?

Question 3-2: Is there any energy loss in this experiment? (*If some of the energy is lost, the experimental final velocity will be below the theoretically predicted values. If the experimental final velocity is larger, what is the significant reason? You can't create energy from nothing!*)

Insightful Activity:
Suppose you use the same shape of object, such as sphere. Prove or falsify the following proposition:

Any size of the object can have an equal final velocity on the same inclined plane.

Experiment with two different spheres to measure the falling time. Do the results prove the above proposition? Check one of the following in accordance with your experimental result.

Proved ____________ Falsified ______________

The evidence:

Check with your TA.

Conceptual Question 1:
Compare the final velocities of each case. Which object can be the fastest? Does it make sense in terms of each moment of inertia? How about acceleration for each case?

Conceptual Question 2:
What is the significant difference between this lab and the Work-Energy Theorem lab? (If you used a cubic object that has the same mass as one of the above objects in this lab, assuming no friction when the cubic object slides, would you be able to obtain the same result?)